

# The relaxation of initial condition in systems with infinitely many absorbing states

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We have investigated the effect of the initial condition on the spreading exponents of the one-dimensional pair contact process (PCP) and threshold transfer process (TTP). The non-order field was found to exhibit critical fluctuations, relaxing to its natural value with the same power-law as the order parameter field. We argue that this slow relaxation, which was not taken into account in earlier studies of these models, is responsible for the continuously changing survival probability exponent. High precision numerical simulations show evidence of a (slight) dependence of the location of the transition point on the initial concentration, in the case of PCP. The damage spreading (DS) point and the spreading exponents coincide with those of the ordinary critical point in both cases.

## I. INTRODUCTION

Recently the question was addressed whether one can construct initial states that affect the *entire* temporal evolution of critical non-equilibrium systems. This is the case of systems that display a phase transition between an active state and a phase with infinitely many absorbing states. Non-universality of dynamic properties, associated with the initial configuration dependence of the survival probability of clusters started from a single active site, has been reported [1–3].

Similar changes in the critical spreading behaviour have been observed by Grassberger et al [4] in a model where long-time memory effects are explicitly introduced. In the system studied by these authors, the susceptibility to the spreading of an active agent changes after the first encounter, remaining constant afterwards. Despite the observed non-universal dynamical critical behaviour, susceptibility in the first encounter does not affect the critical point location. Grassberger et al argue that these results apply to models with multiple absorbing states where an effective memory-dependent susceptibility is present. However, different behaviour is predicted in the case of slowly decaying memory effects, in which case both the value of the critical point and the exponents are expected to depend on the initial state.

A dependence on the initial configuration has also been found [5] in the case of long-range spatial correlations, in which case the dynamical critical exponents change continuously as a function of initial correlation length.

The critical relaxation from an initial homogeneous state in systems with multiple absorbing states was recently examined from a field theoretical (Langevin equations) approach [6]. The evolution equation for the order parameter density was found to include a memory term which is not present in the simpler case of directed percolation (DP).

In the present study we have investigated memory effects in two 1-d models with multiple absorbing states, the PCP [7,1] and TTP [2] models. In both models there is a non-order parameter field, dynamically coupled to the order parameter field, that gives rise to an effective susceptibility for spreading. We show that the non-ordering field relaxes to its steady state value by the same power-law time dependence as the order parameter field and this is characterised by the natural, long-time behaviour exponent of the density decay of a DP process. This is clear evidence of slowly decaying memory. This power-law boundary condition (in time) is similar to that of the long-range power-law boundary condition (in space) of [5] and we can see the emergence of continuously changing dynamical critical exponents. The small shift of the critical point as a function of initial conditions, shown by high precision simulations for the PCP model, is in agreement with the above arguments.

Damage spreading (DS) simulations invented in biology [8] and later in physics [9] are useful to show the stability of the systems with respect to small perturbations. The spreading behaviour has been shown to be sensitive to the dynamics leading to the same steady states. An "objective" definition of DS has been proposed [10] according to which the phase diagrams of the steady states of non-equilibrium models can be divided to sectors in which all, none or parts of the physically possible dynamical rules generate stable damages. The phase transitions between the phases can be continuous and usually belong to the DP universality class [11]. However, if the damage variables possess conservation and the DS absorbing states exhibit symmetries - which usually happens when the DS transition point coincides with the ordinary critical point the DS transition can belong to a different universality class [12,13]. The DS transition cannot be in the passive phase of the replicas, but if it occurs in the active phase, the fluctuating replicas at the DS absorption point exclude the non-DP DS behaviour [13]. We have investigated the DS properties of the

PCP and TTP models; the DS transition is shown to coincide with the ordinary critical point, and the non-universal spreading exponents have been inherited as well.

In section *II*, we give a brief introduction to the PCP and TTP models. Time-dependent and critical relaxation studies are described in *III*, whereas section *IV* is devoted to damage spreading simulations. Some comments and conclusions are presented in *V*.

## II. THE PCP AND TTP MODELS

Both of these models have a single control parameter  $p$  and qualitatively similar phase diagrams, displaying an 'active state' ( $p < p_c$  in case of PCP,  $p > p_c$  for TTP) and (infinitely many) absorbing phases ( $p > p_c$  in case of PCP,  $p < p_c$  for TTP).

In the TTP model, each site may be vacant, single or doubly (active) occupied, and this can be described by a 3-state variable  $\sigma_i = 0, 1, 2$ . In each time step, a site is chosen at random. In the absence of active sites, the dynamics is indeed trivial: if  $\sigma_i(t) = 0$  (or 1), then  $\sigma_i(t+1) = 1(0)$  with probability  $p$  (or  $1-p$ ). The system relaxes exponentially to a steady state where a fraction  $p$  of sites have  $\sigma_i = 1$  and the others are vacant. If  $\sigma_i(t) = 2$ , then  $\sigma_i(t+1) = 0$ ,  $\sigma_{i-1}(t+1) = \sigma_{i-1}(t) + 1$ ,  $\sigma_{i+1}(t+1) = \sigma_{i+1}(t) + 1$  if  $\sigma_{i+1}(t)$  and  $\sigma_{i-1}(t)$  are both  $< 2$  and  $\sigma_i(t+1) = 1$  if only one of the nearest neighbours of site  $i$  ( $j = i-1$  or  $i+1$ ) has  $\sigma_j(t) < 2$ , in which case  $\sigma_j(t+1) = \sigma_j(t) + 1$ . As can be easily seen, the number of active sites either decreases or remains the same in all processes other than  $(1, 2, 1) \rightarrow (2, 0, 2)$ ; the frequency of these processes depends on the concentration of '1'-s, which is controlled by the parameter  $p$ . Any configuration consisting of only '0'-s or '1'-s is absorbing in what concerns the active sites. The absorbing states in this model are fluctuating - in the respective sector of phase space, ergodicity is not broken.

As we show below, the dynamics of '1'-s is however strongly affected by the presence of active sites. At the critical point, the concentration of '1'-s relaxes to its steady state value (equal to  $p_c$ ) by a power-law.

The PCP is a 2-state variable model with multiple absorbing states, each one of them completely frozen in time, contrary to what happens in the TTP. In the PCP, nearest-neighbour pairs of particles (dimers or active sites) annihilate each other with probability  $p$  or create, with probability  $1-p$ , a particle at one of the adjacent (vacant) sites to the dimer. Dimers cannot be generated spontaneously and therefore play the role of the '2'-s in the TTP. There is a natural configuration to which the system at criticality evolves after all activity has died out; the relationship between the natural particle density and  $p_c$  is not a simple one unlike the TTP case. As shown below, the relaxation to this natural state is a slow process - a power-law in time is also found in this case and slowly-decaying memory effects arise as a result of the coupling between the local density of dimers and the local density of isolated occupied sites.

## III. TIME DEPENDENT SIMULATION RESULTS

Time dependent simulations [14] have become an effective tool to explore dynamical critical exponents of systems at non-equilibrium phase transition points. The simulations are started from a single active seed embedded in a sea of inactive sites and followed up to some  $t_{MAX}$  time such that the cluster size can not exceed the system size  $L$ . The quantities usually investigated are the mean number of active sites  $N(t)$  (pairs in case of PCP and '2'-s in case of TTP model) averaged over all trial samples, the survival probability  $P(t)$  of the clusters and the mean spreading size  $R(t)$  of the surviving clusters. At the critical point and for asymptotically long times these quantities exhibit power-law behaviour like

$$N(t) \propto t^\eta \quad (1)$$

$$P(t) \propto t^{-\delta} \quad (2)$$

$$R(t) \propto t^{z/2}, \quad (3)$$

which define the exponents  $\eta$ ,  $\delta$  and  $z$  respectively. The cluster size exponent  $z$  characterising the linear scale is related to the anisotropy exponent of the system  $Z = \nu_{||}/\nu_{\perp}$  by  $Z = 2/z$ . The order-parameter density inside the surviving clusters can be expressed in terms of these exponents as

$$\rho(t) \propto t^{\eta+\delta-dz/2} \quad (4)$$

and is expected to show the same long-time decay as from an arbitrary bulk configuration with  $\rho(0) \neq 0$ , i.e.

$$\rho(t) \propto t^{-\beta/\nu_{||}} \quad (5)$$

where  $\beta$  is the steady state order-parameter exponent.

Exponents  $\eta$  and  $\delta$  are found to depend on the initial concentration of particles  $\rho_1(0)$ , and are related to static exponents  $\beta$  and  $\nu_{||}$  by the hyperscaling relation [2]

$$2\eta + 2(\delta + \beta/\nu_{||}) = dz. \quad (6)$$

Seed growing simulations for the PCP and TTP models have been carried out up to  $t_{MAX} = 8000 - 16000$  time steps for  $2 \times 10^5$  trial runs. We measured the order parameter density  $\rho_2(t)$  as well as the relaxation of the non-order field density  $\rho_1(t)$  towards the natural values  $\rho_1^{nat} \equiv \rho_1(\infty)$  of the models. In case of the PCP we used  $\rho_1^{nat} = 0.242(1)$  [15] while for the TTP model  $\rho_1^{nat} = p_c = 0.6894(3)$  [2]. The densities were measured inside the "infected" regions of surviving clusters only. To estimate the critical exponents and the transition points together, we determined the local slopes of the scaling variables. For example, in the case of the order parameter density we computed

$$-\alpha(t) \equiv \frac{\log[\rho_2(t)/\rho_2(t/m)]}{\log(m)} \quad (7)$$

with  $m = 8$ . When  $p = p_c$ , one should see a horizontal straight line as  $1/t \rightarrow 0$ . The off-critical curves should possess curvature: curves corresponding to  $p > p_c$  should veer upward, curves with  $p < p_c$  should veer downward.

Figures 1 and 2 show the local slopes of  $\Delta\rho_1(t) \equiv \rho_1(t) - \rho_1^{nat}$  for the PCP in case of  $\rho_1(0) = 0$  and  $\rho_1(0) = 0.432$  respectively. For the order parameter density we obtained the same results within numerical accuracy. As one can read off, the particle density exhibits long-time power-law behaviour with DP exponent, but the critical point is slightly lower in case of  $\rho_1(0) = 0$  than in case of  $\rho_1(0) = 0.432$ . This however agrees with the slow-relaxing susceptibility

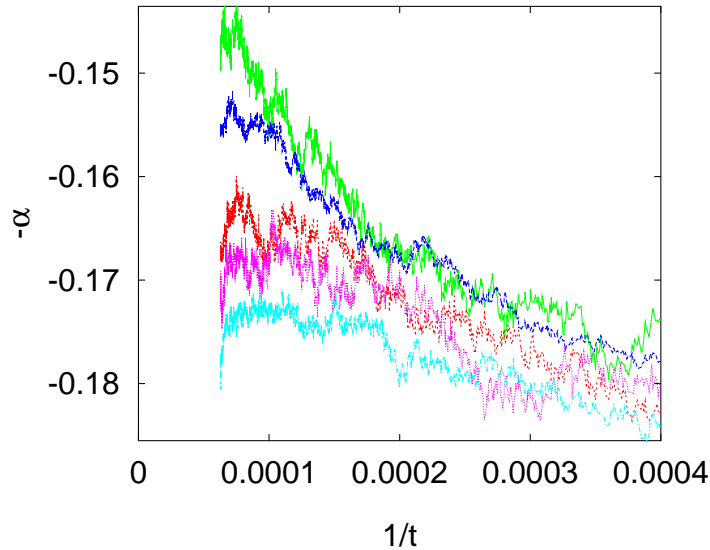


FIG. 1. Local slopes of  $\Delta\rho_1(t)$  in PCP simulations at  $\rho_1(0) = 0$  for  $p = 0.077, 0.07695, 0.0769, 0.07683, 0.07675$  (from bottom to top curves). Scaling can be observed for  $p = 0.07685(5)$ , with the exponent  $\alpha(t \rightarrow \infty) \approx 0.16$ .

picture, because  $\rho_1(0) = 0$  is smaller than the natural value  $\rho_1^{nat} = 0.242(1)$  and increases very slowly in the bulk, therefore a smaller annihilation probability is enough to drive the system to the absorbing state. This dependence can also be the result of the lattice version of the PCP model, where, in contrast to the field theory where there is always some finite density of fields, the isolated '1's are frozen. Both densities exhibit the same exponents in good agreement with  $\beta/\nu_{||} = 0.1596(4)$  of the  $1 + 1$  d DP class. The survival probability exponent measured in our high precision simulation is in agreement with the value of [1] within numerical accuracy.

In the critical point estimates we could see a tiny difference depending on whether we measured the survival probability or the density in the same simulation. Namely the  $p_c$  appears to be nearer to the results of [1,17] if we estimated it from the  $P(t)$  data ( $p_c = 0.07704$  instead of  $p_c = 0.07685$  in case of  $\rho_1(0) = 0$  and  $p_c = 0.07714$  instead

of  $p_c = 0.0770$  in case of  $\rho_1(0) = 0.432$ ). This small offset can be understood on the basis of our density measuring method plus the slow, frozen relaxation in PCP that seems to cause a crossover effect. The density has been averaged in the infected regions, where frozen "islands" can appear which don't evolve at all but keep the non-natural densities for long times. Therefore we overestimate the size of the region in which densities really relax. To verify this picture we have performed simulations where we averaged the '1's and '2' over fixed size. In this case we started the PCP process from randomly distributed pairs in a system of size  $L = 16000$  and followed the evolution up to  $t_{MAX} = 16000$ . In cases of  $\rho_1(0) = 0$  and  $\rho_1(0) = 0.33$  we found that the critical point is about  $p_c = 0.07708$  nearer to the  $P(t)$  and the  $N(t)$  DS results. The scaling exponent was again DP like :  $\alpha \approx 0.16$ .

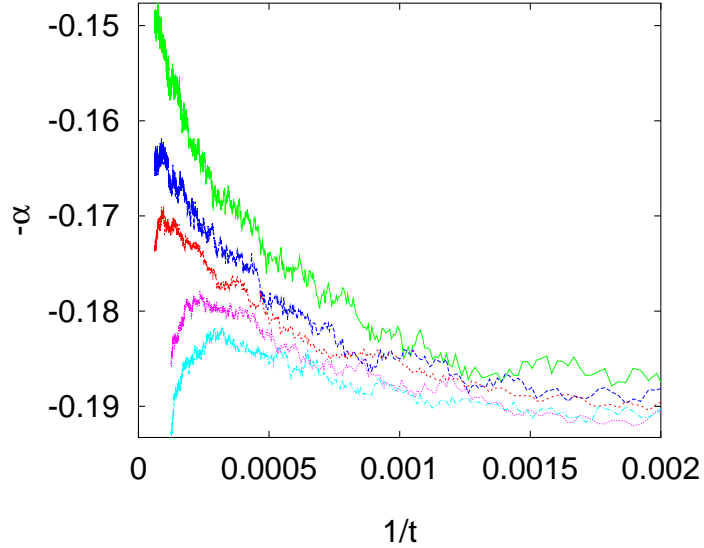


FIG. 2. The same as Figure 1 in case of  $\rho_1(0) = 0.432$  and  $p = 0.07715, 0.0771, 0.07705, 0.077, 0.0769$  (from bottom to top curves). Scaling can be observed for  $p = 0.0770(5)$ , with the exponent  $\alpha(t \rightarrow \infty) \approx 0.16$ .

In the case of TTP model we have performed simulations for  $\rho_1(0) = 0.4$ ,  $\rho_1(0) = 0.6894$  and  $\rho_1(0) = 0.8$  initial particle densities. In this model, we did not observe any shift of  $p_c$ , as Figure 3 shows, and all the densities scale with the  $\beta/\nu_{||} = 0.1596(4)$  exponent.

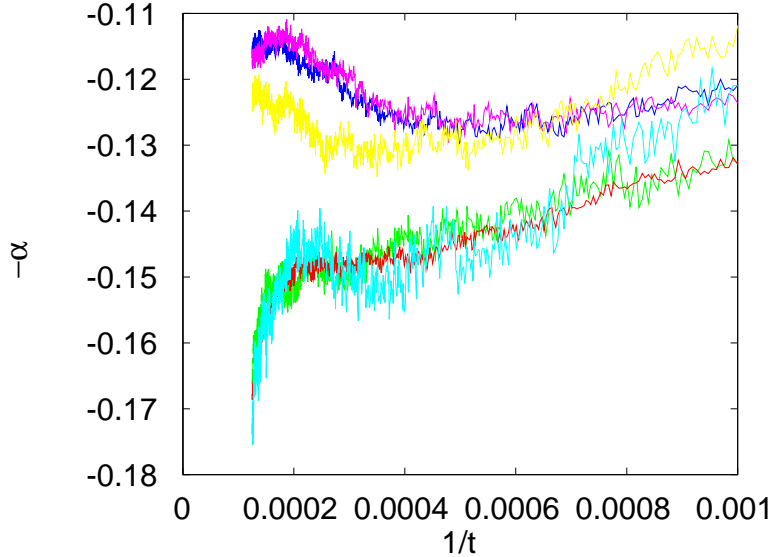


FIG. 3. Local slopes for  $\Delta\rho_1(t)$  in TTP simulations for  $p = 0.6887$  (top curves),  $p = 0.6897$  (bottom curves); and  $\rho_1(0) = 0.4, 0.6894, 0.8$ . Scaling can be observed for about  $p = 0.6894$ , with the exponent  $\alpha(t \rightarrow \infty) \approx 0.16(1)$ .

The similar power-law behaviours of  $\rho_2(t)$  and  $\Delta\rho_1(t)$  can be understood if one considers the coupled Langevin

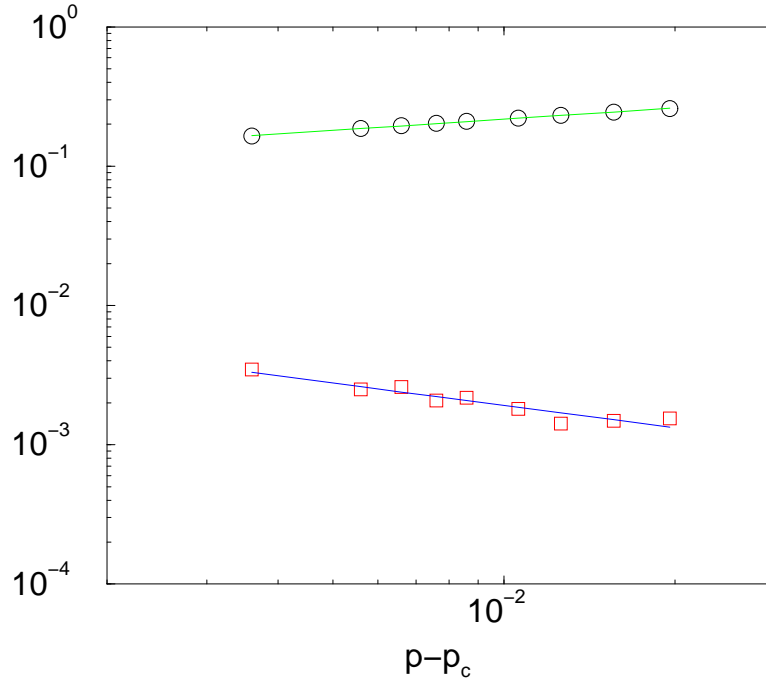


FIG. 4. Log-log plot of  $\rho_1(p) - \rho_1(p_c)$  ( $\circ$ ) and respective fluctuation ( $\square$ ) v.s.  $p - p_c$  above the critical point of the TTP model. Least-square regression results in  $\beta = 0.27(2)$  and  $\gamma = 0.53(6)$ . The averaging was done following 45000 initialisation MC steps over 5000 time steps and 300 trials.

equations describing the time evolution of the processes. For example, in case of the PCP model, they look like [6]:

$$\frac{\partial \phi_1(x, t)}{\partial t} = c_1 \nabla_x^2 \phi_2 + r_1 \phi_2 - u_1 \phi_2^2 - w_1 \phi_1 \phi_2 + \dots + \eta_1(x, t) \quad (8)$$

$$\frac{\partial \phi_2(x, t)}{\partial t} = c_2 \nabla_x^2 \phi_1 + r_2 \phi_1 - u_2 \phi_1^2 - w_2 \phi_1 \phi_2 + \dots + \eta_2(x, t) \quad (9)$$

where  $\eta_1(x, t)$  and  $\eta_2(x, t)$  are Gaussian uncorrelated noise terms proportional to  $\sqrt{\phi_2}$ . One can see that the equations are coupled strongly by  $w$ -terms; since the right hand sides of the equations contain the same powers of the scaling fields, the time derivatives are expected to have the same scaling too.

In reference [6] it was argued that

$$\phi_1(t) = \phi_1^{nat} + (\phi_1(0) - \phi_1^{nat}) e^{-w_1 \int_0^t \phi_2(x, s) ds} \quad (10)$$

( $\phi_1^{nat} = r_1/w_1$  is the natural concentration in this formalism) may be taken as an approximate solution of equation (8), in which case the  $\phi_1 \phi_2$  cross-term in (9) has the form

$$-w_2 \phi_2 (\phi_1(0) - \phi_1^{nat}) e^{-w_1 \int_0^t \phi_2(x, s) ds}. \quad (11)$$

The power-law time dependence of  $\phi_1$  is obvious, because an exponential relaxation to  $\phi_1^{nat}$  would just give in (9) a term similar to  $r_2 \phi_2$  that can shift the critical point but not the critical indices (from the DP values).

The long-range scaling behaviour of the non-order density suggests that the  $\phi_1$  field possesses critical fluctuations. To test this, we have performed steady state simulations as well. We have measured the '0', '1' and '2' densities in case of the TTP model just above the critical point. We considered  $L = 4000$  systems and let them evolve from random initial conditions with  $p$  slightly above  $p_c = 0.6894$ ; about 40000 MC lattice updates were necessary to reach the steady state. As one can see in Figure 4, least-square fits of  $\log(\rho_1(p) - \rho_1(p_c))$  v.s.  $\log(p - p_c)$  resulted in regular DP scaling exponent  $\beta = 0.27$  [16]. The other two densities ('0'-s and '2'-s) exhibited the same steady state exponents too. For the fluctuation of  $\Delta \rho_1 \equiv \rho_1(p) - \rho_1(p_c)$  we observed the scaling

$$\langle \Delta \rho_1^2 \rangle - \langle \Delta \rho_1 \rangle^2 \propto |p - p_c|^{-\gamma} \quad (12)$$

with  $\gamma = 0.53(6)$  exponent, which agrees with DP universality class value again [19].

The critical behaviour of the  $\phi_1$  field demands that extra care is taken when dealing with truncated versions of equations (8) and (9). A numerical integration of (9), including the non-Markovian term (11), was carried out by Lopez and Muñoz [20] and revealed that the presence of the memory term is responsible for scaling up to some time with non-universal values of  $\eta$  and  $\delta$ . However, these authors did not find the linear relation between the shift from the DP values ( $\eta - \eta_{DP}$  and  $\delta - \delta_{DP}$ ) and  $\phi_1(0) - \phi_1^{nat}$  that our results for PCP show (Figure 5) and was also found in previous TTP studies [18]. We think one has to take into account the omitted terms in eq. (10), which we have shown to exhibit power law in time and therefore give a relevant contribution to the renormalization of  $\phi_2$ .

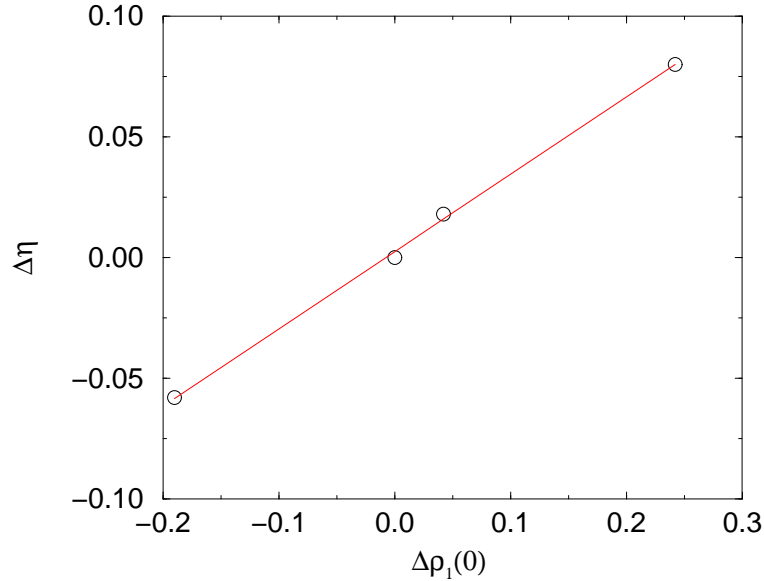


FIG. 5. Initial concentration dependence of the exponent  $\eta$  for PCP model. Linear regression gives a slope  $0.320(7)$  between  $\eta - \eta_{DP}$  and  $\rho_1(0) - \rho_1^{nat}$

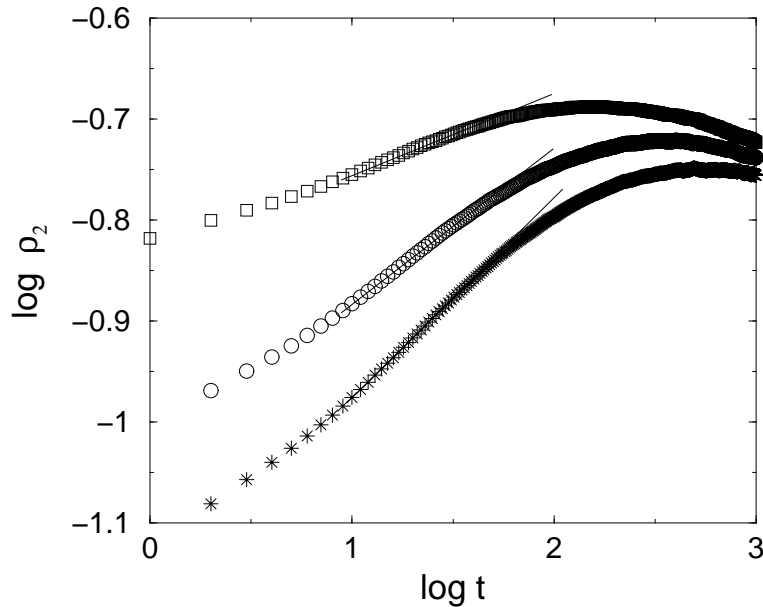


FIG. 6. Double-logarithmic plot of  $\rho_2$  v.s.  $t$  for critical TTP in early times for  $\rho_2(0) = 0.02$  and  $\rho_1(0) = 0.4, 0.69$  and  $0.8$  (top to bottom curves). The slopes of the straight lines are  $0.08, 0.15$  and  $0.19$  (respectively).

We have also investigated the early stages of the relaxation of the order parameter. In reference [6] it was suggested that in the short time regime ( $w_1\phi_2t \ll 1$ ) one might observe the dynamic percolation

scaling of Grassberger [4]. Early time scaling - currently referred to as *critical initial slip* - was introduced by Jansen et al [21] and recently investigated by van Wijland et al [22] for a reaction-diffusion model with two kinds of particles, using RG analysis. They found non-universal dependence (on the initial particle distribution) of the slip exponent in the case of unequal diffusion coefficients.

The system was prepared by 'adding' a few '2'-s (uniform density  $\rho_2(0) \ll 1$ ) to a random uniform background of '0'-s and '1'-s (density  $\rho_1(0)$ ). The system evolution was recorded up to  $t = 1000 MCS$  and averages were performed over independent runs ( $10^4$  typically). Log-log plots of  $\rho_2(t)$  - see Fig. 6 - show a linear region (lasting for  $10 < t < 100$  for  $\rho_2(0)$  equal to a few percent) with a slope that is independent of  $\rho_2(0)$  but depends on  $\rho_1(0)$ . This may be evidence for the critical initial slip with a non-universal slip exponent  $\theta'$  equal to that slope. Unlike the DP case, where  $\theta' = \eta$  [22], the values we found for  $\theta'$  differ significantly from the corresponding  $\eta$  values.

#### IV. DAMAGE SPREADING SIMULATIONS

The damage spreading simulations have been initialised by two replicas of states with identical, but random uniform distribution of single '1'-s of given concentrations. Then a seed (a pair in case of PCP and a '2' in case of TTP) is added to each replica such that they become nearest neighbours and the initial difference is 2 (see fig.7). The order

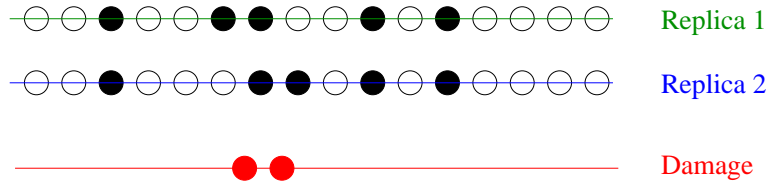


FIG. 7. Sample initial state of PCP DS simulations. Pairs (seeds) are displaced in the middle of the lattices of replicas with a single space shift that generates 2 damage variables. In case of TTP DS simulations the initialisation is the same, except that we have '2'-s instead of pairs.

parameter characterising the damage is the Hamming distance between replicas

$$D(t) = \left\langle \sum_{i=1}^L |s(i) - s'(i)| \right\rangle . \quad (13)$$

where  $s(i)$  denote the pairs in case of PCP and the variable '2'-s in case of the TTP model. At the DS critical point ( $p_d$ ) we expect that the order parameter scales as

$$D(t) \propto t^\eta , \quad (14)$$

Similarly the survival probability of damage variables behaves as

$$P(t) \propto t^{-\delta} \quad (15)$$

and the average mean square distance of damage spreading from the center scales as

$$R^2(t) \propto t^z . \quad (16)$$

Averages were performed over  $N_s = 10^6$  independent runs for each value of  $p$  in the vicinity of  $p_d$  ( but for  $R^2(t)$  only over the surviving runs). The  $t_{MAX}$  was 8000 in these simulations. Figures 8, 9, 10 and 11 show the local slopes results of the Hamming distance for the PCP model and different initial concentration of '1'-s. The  $p_d$  transition points are found to coincide with the ordinary critical points of the replicas within numerical accuracy. A small, but monotonic tendency in the variation of  $p_d$  (as in case of  $p_c$ ) with initial conditions can be observed in Table I:

The DS critical  $\eta$  and  $\delta$  exponents show the same non-universal behaviour as the corresponding ordinary critical exponents (see also [15,17]) and coincide with them within numerical precision. The exponent  $z$  is constant within numerical accuracy.

In the case of  $\rho_1(0) = 0.242$ , we performed runs with uniform initial distributions and with system generated configurations of isolated '1'-s with the same average concentration. The latter, non-uniform distribution was generated by letting a single replica to run at  $p = 0.07709$  until it reached the absorbing state; then the infected area of the

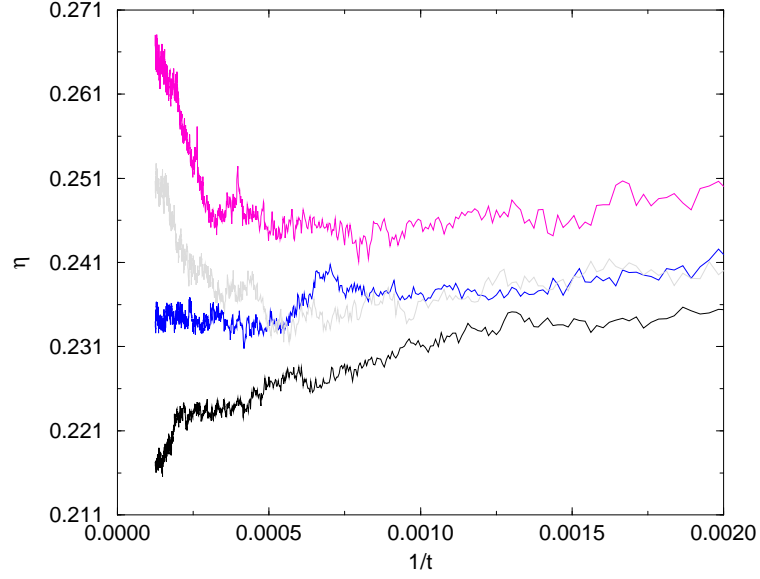


FIG. 8. Local slopes  $\eta(t)$  near the PCP DS transition point, for  $\rho_1(0) = 0$  and  $p = 0.07708, 0.07704, 0.077, 0.0769$  (from bottom to top). The DS critical point is at  $p_d = 0.07704$  with the corresponding exponent  $\eta = 0.234(3)$ .

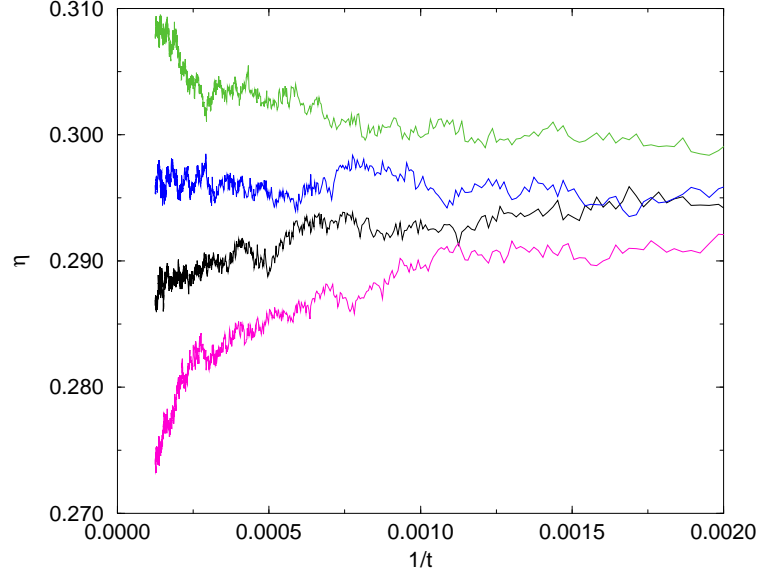


FIG. 9. Same as figure 8 for  $\rho_1(0) = 0.2$  and  $p = 0.07716, 0.07712, 0.07708, 0.07704$  (bottom to top). The DS critical point is at  $p_d = 0.07708$  with the corresponding  $\eta = 0.296(1)$ .



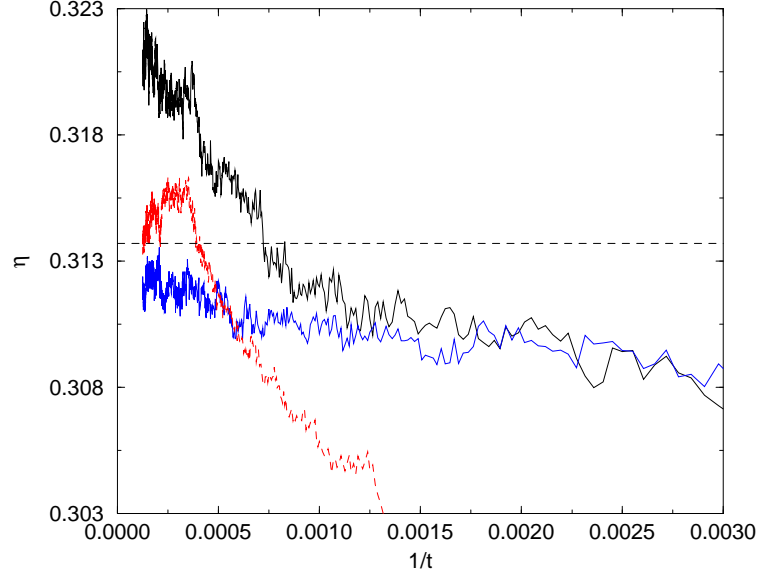


FIG. 10. Same as figure 8 for  $\rho_1(0) = 0.242$  and  $p = 0.07709, 0.07707$  (from bottom to top). The dotted curve corresponds to simulations with system generated initial configurations and  $p = 0.07709$ . The DS critical point is at  $p_d = 0.07709$  with the corresponding exponent  $\eta = 0.314(6)$ . The dashed line shows the estimated value of the DP exponent obtained by simulations.

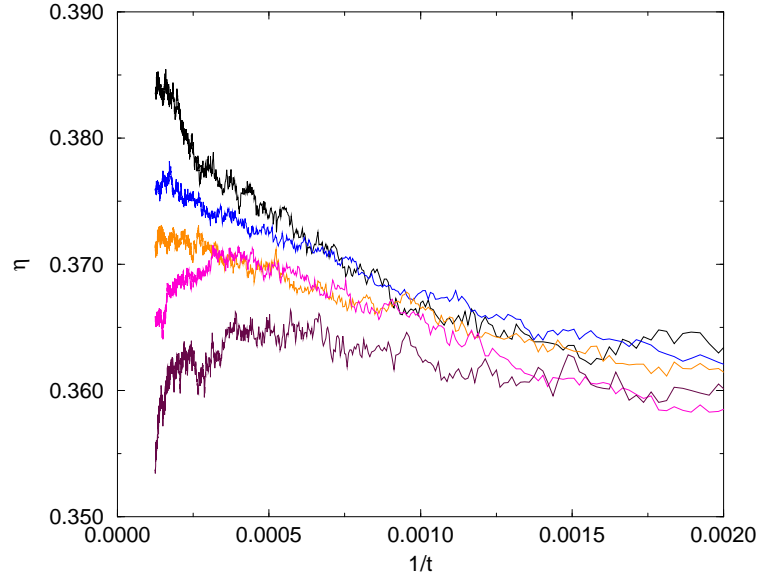


FIG. 11. Same as figure 8 for  $\rho_1(0) = 0.432$  and  $p = 0.07718, 0.07716, 0.07712, 0.0771$  (from bottom to top). The DS critical point is at  $p_d = 0.07714$  with the corresponding exponent  $\eta = 0.372(5)$ .

$\rho_1(0)$	$p_d$	$\eta_{DS}$	$\delta_{DS}$	$z$
0.0	0.07704	0.234(3)	0.24(2)	1.23(3)
0.2	0.07708	0.296(1)	0.18(1)	1.24(4)
0.242	0.07709	0.314(6)	0.16(9)	1.24(5)
0.432	0.07714	0.372(5)	0.11(1)	1.26(3)

TABLE I. *Damage spreading simulation results in the PCP model*

system was used as initial state for DS simulations, similarly to what was done in ref. [1]. We don't see significant differences (Fig. 10) between the two cases for  $t$  large, both of them result in exponents in agreement with the best DP class  $\eta_{DP} = 0.3137$  value [19].

In case of the TTP model we performed DS simulations for  $\rho_1(0) = 0.4$  only. We could see analogous DS behaviour as in the case of PCP. Again the critical point and exponents coincide with the corresponding critical values.

Similarly to what has been observed in models that belong to the PC universality class [13], we can also conclude that if the DS transition point coincides with  $p_c$  the scaling behaviour is inherited.

An interesting implication of this result can be stated exploiting the possible mapping of these models to SOC models [15]. The corresponding critical sandpile models are not chaotic in the sense that the avalanches (or clusters) arising from the dropping of pairs (or seeds) to the lattice result in trajectories with power-law increasing differences only. This does not exclude the possibility that other SOC models generated by the way of ref. [15] are chaotic, since if  $p_d$  happens to be in the active phase, the perturbations in the SOC model generate differences that increase faster than a power law.

## V. CONCLUSIONS

Two representatives of systems with a continuous phase transition to infinitely degenerate absorbing states (PCP and TTP models) have been investigated numerically in  $1-d$ . In order to clarify the influence of the initial condition on the dynamic properties, we have performed time-dependent simulations and analysed the evolution of both the order parameter and the non-order field densities. We gave numerical evidence that the non-order field is in a critical state simultaneously with the order parameter field. The isolated particles density exhibits a continuous phase transition with DP exponents to a non-absorbing state, therefore its fluctuations cannot be neglected when one tries to understand the non-universal behaviour of critical exponents. Due to the dynamic coupling between the two fields, the slow (power-law) decay of the background particle density induces a long-time memory of the susceptibility to spreading of the order parameter [4].

We have found that the estimates of  $p_c$  obtained from the time dependence of the survival probability are consistent with those given by DS studies; a small shift as a function of  $\rho_1(0)$  is exhibited, which we interpret as due to the slowly decaying memory [4]. The study of the density in seed-growing simulations produced values of  $p_c$  slightly off the former ones; we think this is probably a crossover effect.

We don't see such a  $p_c$  shift for the TTP model so this maybe specific of the PCP model, connected to the non-ergodicity of its absorbing states. In both models the critical exponents  $\eta$  and  $\delta$  have been found to behave linearly as a function of the initial particle concentration.

Preliminary studies of the early time critical regime suggest the existence of a (short) initial slip regime characterised by an exponent which depends on the initial particle concentration  $\rho_1(0)$ , in agreement with RG predictions for a similar model [22]. This topic needs however additional investigation.

The damage spreading investigations have shown that the DS point coincides with the critical point and so the critical indices "inherit" the same non-universal scaling behaviour similarly what was found in an earlier study [13].

We hope that our study will stimulate further field theoretical analysis of critical spreading in systems with many absorbing states.

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